

(3 Hours)

[Total Marks : 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Question Nos. 2 to 6.

1. (a) Evaluate $\int_C (z - z^2)$ where C is the upper half of the circle $|z| = 1$. What is the value of the integral for the lower half of the same circle? 5

(b) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$. Find the eigen values of $A^3 + 5A + 8I$. 5

(c) The regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$. Find (1) mean of x and y and (2) coefficient of correlation between x and y. 5

(d) A machine is claimed to produce nails of mean length 5 cm. and standard deviation of 0.45 cm. A random sample of 100 nails gave 5.1 cm. as average length. Does the performance of the machine justify the claim? Mention the level of significance you apply. 5

2. (a) Show that the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory. 6

(b) Evaluate $\int \frac{z+3}{z^2+2z+5} dz$, where C is the circle (i) $|z| = 1$. (ii) $|z+1-i| = 2$. 6

(c) The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 cm and the standard deviation is 0.005 cm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 cm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed. 8

3. (a) A continuous random variable X has the following probability law $f(x) = kx^2 e^{-x}$, $x \geq 0$. Find k, mean and variance. 6

(b) Solve the following LPP by Simplex method :— 6

$$\begin{aligned} \text{Max } z &= x_1 + 4x_2 \\ \text{Subject to } 2x_1 + x_2 &\leq 3 \\ 3x_1 + 5x_2 &\leq 9 \\ x_1 + 3x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(c) Find Laurent's series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ when 8

$$(i) |z| < 1 \quad (ii) 1 < |z| < 2 \quad (iii) |z| > 2.$$

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4. (a) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sums of the squares of the deviation from the means are 26.94 and 18.73 respectively. Can the samples be considered to have been drawn from the same population? 6
- (b) Calculate the correlation coefficient from the following data : 6
- X : 23 27 28 29 30 31 33 35 36 39
- Y : 18 22 23 24 25 26 28 29 30 32
- (c) Show that the following matrix is Diagonalizable. Find the transforming matrix and the Diagonal matrix. 8

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

5. (a) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6
- (b) Evaluate the following integral by contour integration 6

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)}$$

- (c) Use Kuhn Tucker method to solve the NLPP :— 8

$$\text{Max } Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$$

$$\text{St } x_1 + x_2 \leq 2$$

$$2x_1 + 3x_2 \leq 12$$

$$x_1, x_2 \geq 0.$$

6. (a) For special security in a certain protected area, it was decided to put three lighting bulbs on each pole. If each bulb has a probability p of burning out in the first 100 hours of service, calculate the probability that at least one of them is still good after 100 hours. 6

If $p = 0.3$, how many bulbs would be needed on each pole to ensure 99% safety that atleast one is good after 100 hours ?

- (b) Use Duality to solve the following LPP : 6

$$\text{Max } Z = 2x_1 + x_2$$

$$\text{Subject to } 2x_1 - x_2 \leq 2$$

$$x_1 + x_2 \leq 4$$

$$x_1 \leq 3$$

$$x_1, x_2 \geq 0$$

- (c) The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use χ^2 test to check whether these frequencies are in agreement with the belief that occurrence of accidents was the same during 10 months period. Test at 5% level of Significance. 8